INDIAN INSTITUTE OF MANAGEMENT & COMMERCE



DEGREE & P.G COLLEGE, An Autonomous College

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QUESTION BANK

SUBJECT: STATISTICS

PAPER: BASIC STATISTICS AND PROBABILITY

UNIT-I

Basic Statistics:

- 1. Explain various Measures of Dispersion.
- 2. The average salary of male employees in a firm was Rs. 5200 and that of female was Rs. 4200. The mean salary of all the employees was Rs. 5000. Find the percentage of male and female employees.
- 3. Define absolute and relative measure of dispersion and give two examples each.
- 4. The growth rate of population of a country in three successive years was 3.0%, 2.5% and 2.1% respectively. Find the average population growth of country.
- 5. Let the geometric means of three groups contains 8, 7 and 5 observations are 8.52, 10.12 and 7.75 respectively. Then find the geometric mean of the 20 observations in the single group formed by the pooling the three groups.
- 6. A man travels from Karimnagar to Adilabad by a car and takes 4 hours to cover the whole distance. In the first hour he travels at a speed of 50 km/hr, in the second hour his speed is 64 km/hr, in the third hour his speed is 80 km/hr and in the fourth hour he travels at the speed of 55 km/hr. find the average speed of the traveller?
- 7. A cyclist pedals from his house to his college at a speed of 10 km.p.h. and back from the college to his house at 15 km.p.h. find the average speed.
- 8. Define Central moments. Derive the relations of Central moments in terms of Non central moments.
- 9. Define Non-central moments. Establish the relationship between Non central moments in terms of Central moments.
- 10. The first four moments of a distribution about the value 4 of the variable are -1.5, 17, -30, and 108. Find the moments about mean, β_1 and β_2 .
- 11. The first four moments of a distribution about the value 5 of the variable are -4, 22, -117, and 560. Find the mean, moments about mean, β_1 and β_2 .
- 12. Explain Sheppard's Correction for Moments.
- 13. Show that Central moments are independent of change of origin but dependent on change of scale.
- 14. Show that Non central moments are independent of change of origin but dependent on change of scale.
- 15. Define Skewness and Explain various Measures of Skewness.
- 16. Define various types of Skewness.
- 17. Derive the limits of Bowley's Coefficient of Skewness.
- 18. Derive the limits of Karl Pearson's Coefficient of Skewness.
- 19. A frequency distribution gave the following results: C.V = 5, (ii) Karl Pearson's Skewness = 0.5 (iii) S.D = 2, Find the mean and mode of the distribution.

- 20. For a distribution Karl Pearson's Coefficient of Skewness is 0.64, standard deviation is 13 and mean is 59.2. Then find mode and median.
- 21. Describe briefly about Kurtosis.
- 22. Write the steps involved in preparing statistical analysis report based on descriptive statistics

UNIT-II

Probability:

- 23. Define Classical definition of probability also write down limitations of Classical definition.
- 24. What is the chance that a leap year selected at random will contain 53 Sundays?
- 25. Define statistical definition of probability.
- 26. What is the probability that four S's come consecutively in the word MISSISSIPPI?
- 27. In a random arrangement of the letters of the word 'MATHEMATICS' find the probability that all the vowels come together.
- 28. Define Axiomatic approach of probability.
- 29. State and prove Addition theorem of probability for 'n' events.
- 30. If two dice are thrown, What is the probability that the sum is neither 7 nor 11?
- 31. An integer is chosen at random from two hundred digits. What is the probability that the integer is divisible by 6 or 8?
- 32. The probability that a student passes a physics test is 2/3 and the probability that he passes both a physics test and an English test is 14/45. The probability that he passes at least one test is 4/5. What is the probability that he passes the English test?
- 33. Three newspapers A, B and C are published in a certain city. It is estimated from a survey that of the adult population: 20% read A, 16% read B, 14% read C, 8% read both A and B, 5% read both A and C, 4% read both B and C, 2% read all three. Find what percentage read at least one of the papers?
- 34. Define Conditional probability.
- 35. State and prove Multiplication theorem of probability for 'n' events.
- 36. From a city population, the probability of selecting (i) a male or a smoker is 7/10, (ii) a male smoker is 2/5, and (iii) a male, if a smoker is already selected is 2/3. Find the probability of selecting (a) a non-smoker, (b) a male, and (c) a smoker, if a male is first selected.
- 37. Define (i) Independent Events (ii) Pairwise Independent Events (iii) Mutually Independent Events.
- 38. If A and B are independent events, then show that A and B^c are also independent.
- 39. If A and B are independent events, then show that A^c and B are also independent.
- 40. If A and B are independent events, then show that A^c and B^c are also independent.
- 41. A problem in statistics is given to three students A, B and C whose chances of solving it are 1/2, 3/4 and 1/4 respectively. What is the probability that the problem will be solved if all of them try independently?
- 42. If P (A \cap B) = 1/8, P (B) = 1/5 and P (A) = 3/4. Then find the value of P (A/B^c)
- 43. If A and B are two events, not mutually exclusive, connected with a random experiment P(A)=1/4; P(B)=2/5 and P(AUB)=1/2. Then find (i) $P(A \cap B^c)$ (ii) $P(A^c \cap B^c)$
- 44. State and prove Boole's inequalities.
- 45. State and prove Baye's theorem.
- 46. The probabilities of X, Y and Z becoming managers are 4/9, 2/9 and 1/3 respectively. The probabilities that the Bonus scheme will be introduced if X, Y and Z become managers are 3/10, 1/2 and 4/5 respectively. (i) What is the probability that Bonus scheme will be introduced, and (ii) If the Bonus has been introduced, what is the probability that the manager appointed was X?
- 47. The contents of Urn I: 1White, 2black and 3red balls

Urn2: 2white, 1black and 1red balls

Urn3: 4white, 5black and 3red balls.

- One urn is chosen at random and 2 balls drawn from it. They happen to be white and red. What is the probability that they come from Urn I.
- 48. A manufacturing company produces pipes in 2 plants I and II with daily production of 1500 and 2000 respectively. The fraction defective of the pipes produced by two plants I and II are 0.006 and 0.008 respectively. If the pipe is selected at random, from the days production and is found to be defective. What is the probability that it has come from plant I.
- 49. A letter is known to have come either from TATANAGAR or from CALCUTTA. On the envelope just two consecutive letters TA are visible. What is the probability that the letter came from CULCUTTA.

UNIT-III

Random Variables:

- 50. Define random variable and give two examples.
- 51. Define Discrete random variable and its probability function.
- 52. A random variable X has the following probability function:

X	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	K^2	$2k^2$	$7k^2 + k$

- (i) Find k, (ii) Evaluate P (X < 6), P($X \ge 6$) and P(0 < X < 5), (iii) Determine the distribution function of X (iv) If P($X \le x$) > 1/2 Find the minimum value of x.
- 53. Define Continuous random variable and its probability function.
- 54. A Continuous random variable X has a probability density function $f(x) = 3x^2$, $0 \le x \le 1$. Find a and b such that (i) P ($X \le a$) = P (X > a), and (ii) P (X > b) = 0.05.
- 55. Metro train arrives punctually at metro station for every 15 minutes. Each morning a software employee leaves his house and casually walks to metro station. Let the random variable X denotes the amount of time in minutes the software employee waits for metro train from the time he reaches the station. It is known that probability density function of X is f(X) = 1/15; 0 < x < 15. Find the average waiting time in metro station.
- 56. The diameter of an electric cable, say X is assumed to be a continuous random variable with probability density function f(x) = 6x(1-x) $0 \le x \le 1$.
 - (i)Check that f(x) is PDF. (ii) Determine a number b such that $P(X \le b) = P(X > b)$, (iii) Find a)Arithmetic Mean, b) Harmonic Mean, c) Mode, d) Median, e) Skewness and Kurtosis
- 57. The length of time (in minutes) that a certain lady speaks on the telephone is found to be random variable, with a probability function specified by the probability density function $f(x) = Ae^{-(x/5)}$, for $x \ge 0$
 - (i) Find the value of A (ii) What is the probability that the number of minutes that she will talk over the phone is: a) more than 10 minutes, b) less than 5 minutes and (c) between 5 and 10 minutes.
- 58. The length (in hours) X of a certain type of light bulb may be supposed to be a continuous random variable with probability density function $f(x) = \frac{A}{x^3}$; 1500 < x < 2500
 - (i)Determine the constant A (ii)Compute the probability of the event 1700 < x < 1900.
- 59. Define Distribution Function. State and prove its properties.
- 60. Define joint probability mass function, Marginal probability mass functions of X and Y and Conditional probability functions of X given Y and Y given X.
- 61. A two-dimensional random variable (X,Y) have a bivariate distribution given by:
 - P (X = x, Y = y) = $\frac{x^2 + y}{32}$, for x = 0,1,2,3 and y = 0,1. Find the marginal distributions of X and Y.
- 62. A two-dimensional random variable (X,Y) have a bivariate distribution given by: $P(X = x, Y = y) = \frac{2x+y}{27}$, for x = 0,1 and 2.; y = 0,1 and 2. Find the conditional distribution of Y for X = x, and X for Y = y.
- 63. Define joint probability density function, Marginal probability density functions of X and Y and Conditional probability density functions of X given Y and Y given X.

- 64. The joint probability density function of two dimensional random variable (X,Y) is given by f(x, y) = 2 where 0 < y < x < 1 then Find (i) the marginal densities of X and Y, (ii) Conditional densities of X given Y and Y given X, (iii) Check for independence of X and Y.
- 65. If X and Y are two random variables having joint density function:

$$f(x,y) = \frac{1}{8}(6 - x - y); 0 \le x < 2, 2 \le y < 4$$
Find (i) P (X < 1, Y < 3), (ii) P (X + Y < 3), and (iii) P(X < 1 / Y < 3).

- 66. Explain transformation of one dimensional random variable.
- 67. The random variable X has an exponential distribution: $f(x) = e^{-x}$, $0 < x \le \infty$. Find the density function of the random variable (i) Y = 3X + 5, (ii) $Y = X^3$
- 68. Let X be a random variable with p.d.f: $f(x) = \frac{1}{2}$; -1 < x < 1. Then find the p.d.f of $Y = X^2$

UNIT-IV

Mathematical Expectation:

- 69. A player tosses 3 coins. He wins Rs. 16, if 3 heads appear, Rs. 8, if 2 heads appear Rs. 4, if 1 head appears Rs. 2 if no head appears. Find his expected amount of winning and variance.
- 70. A person selects two balls at random from the bag containing 4 white and 3 black balls. He receives Rs. 15, if a white ball and Rs. 10 for each black ball selected, then find his expectation.
- 71. If two unbiased dice are thrown. Find the expected values of the sum of numbers of points on them.
- 72. Prove that E (a X + b) = a E (X) + b.
- 73. Prove that V (a X + b) = a^2 V (X).
- 74. State and prove Multiplication theorem of expectation of 'n' independent random variables.
- 75. State and prove Addition theorem of expectation of 'n' random variables.
- 76. State and prove Chebychev's inequality.
- 77. Two unbiased dice are thrown. If X is the sum of the numbers showing up, prove that $P(IX 7I) \le \frac{35}{54}$
- 78. A symmetric die is thrown 720 times. Use Chebychev's inequality to find the lower bound for the probability of getting 100 to 140 sixes.
- 79. If X is a random variable such that E (X) = 3 and E (X^2) = 13, use Chebychev's inequality to determine a lower bound for P (-2 < X < 8).
- 80. State and prove Cauchy-Schwartz inequality.
- 81. Write any two applications on the usage of Cauchy-Schwartz inequality.
- 82. Define Characteristic Function. State and prove its properties.
- 83. Define Moment Generating Function. State and prove its properties.
- 84. Define Cumulant Generating Function. State and prove its properties
- 85. Define Probability Generating Function. State and prove its properties.