UNIT – IV – LAST CONCEPT

Black and Scholes Option Pricing Model (BOPM)

It was initially developed in 1973 by two academicians, Fisher Black & Myron Scholes & was designed to price European options on non-dividend paying stocks.

Also called Black-Scholes-Merton, it was the first widely used model for option pricing. It is used to calculate the theoretical value of options using current stock prices, expected dividends, the option's strike price, expected interest rates, time to expiration and expected volatility.

The formula, developed by three economists—Fischer Black, Myron Scholes and Robert Merton—is perhaps the world's most well-known options pricing model.

The Black-Scholes model makes certain assumptions:

- The option is European and can only be exercised at expiration.
- No dividends are paid out during the life of the option.
- Markets are efficient (i.e., market movements cannot be predicted).
- There are no transaction costs in buying the option.
- The risk-free rate and volatility of the underlying are known and constant.
- The returns on the underlying are log-normally distributed.
- There are no arbitrage opportunities.
- Stock is being traded continuously.
- It is possible to borrow and lend cash at a constant risk-free interest rate.
- The price of the underlying stock moves randomly.
- Short selling of the underlying stock is possible.

The Black Scholes Formula

The mathematics involved in the formula are complicated and can be intimidating. Fortunately, you don't need to know or even understand the math to use Black-Scholes modeling in your own strategies. Options traders have access to a variety of online options calculators, and many of today's trading platforms boast robust options analysis tools, including indicators and spreadsheets that perform the calculations and output the options pricing values.

The Black Scholes call option formula is calculated by multiplying the stock price by the cumulative standard normal probability distribution function. Thereafter, the net present value (NPV) of the strike price multiplied by the cumulative standard normal distribution is subtracted from the resulting value of the previous calculation.

In mathematical notation:

$$\mathbf{C = S \times N(d_1) - Ke^{-rt} \times N(d_2)}$$

where:

$$\mathbf{d1} = \left\{ \boldsymbol{ln}\left(\frac{\boldsymbol{S}}{\boldsymbol{K}}\right) + (\boldsymbol{r} + .5\boldsymbol{\sigma}2)\boldsymbol{t} \right\} \div \boldsymbol{\sigma}\sqrt{\boldsymbol{t}}$$

$\mathbf{d_2 = d_1 -} \ \boldsymbol{\sigma}\sqrt{\boldsymbol{t}}$

where:

C = Call option price/value of a call option

S = Current market price of the underlying shares

K = Strike price of the option

e = Base of Natural Logarithms

r = Risk-free interest rate

t = Time to maturity or expiry

N = Cumulative standard normal distribution

ln = Natural log, i.e., log to the base e.

σ = Annualised Standard deviation of stock returns as a decimal (volatility)

Advantages of Black-Scholes Model

1. The Black Scholes Model is one of the most important concepts in modern financial theory.
2. The Black Scholes Model is considered as the standard model for valuing options.
3. A model of price variation over time of financial instruments such as stocks that can, among other things, be used to determine the price of a European call option.
4. The model assumes that the price of heavily traded assets follow a geometric Brownian motion with constant drift and volatility.
5. When applied to a stock option, the model incorporates the constant price variation of the stock, the time value of money, the option's strike price and the time to the option's expiry.
6. It enables one to calculate a very large number of option prices in a very short time. It works entirely on objective figures rather than human judgment. Fortunately one does not have to know calculus to use the Black Scholes model.

Limitations of Black-Scholes Model

1. Black-Scholes model cannot be used to accurately price options with an American-style exercise as it calculates the option price at expiration only. Early exercise as in the case of American option cannot be priced correctly using this model which is a major limitation of this model.

2. All exchange traded equity options have American-style exercise as against the European options which can only be exercised at expiration. That means this model cannot be used for

pricing most exchange traded options. The exception to this is an American call on a non dividend paying asset as the call is always worth the same as its European equivalent since there is never any advantage in exercising early.

3. It makes many assumptions which are not true in reality like it does not consider transaction costs, dividends on the underlying stock, etc.

The Black–Scholes model disagrees with reality in a number of ways, some significant. It is widely used as a useful approximation, but proper use requires understanding its limitations – blindly following the model exposes the user to unexpected risk.

Among the most significant limitations are:

1. The Black-Scholes Model assumes that the risk-free rate and the stock's volatility are constant.

2. The Black-Scholes Model assumes that stock prices are continuous and that large changes (such as those seen after a merger announcement) don't occur.

3. The Black-Scholes Model assumes a stock pays no dividends until after expiration.

4. Analysts can only estimate a stock's volatility instead of directly observing it, as they can for the other inputs.

5. The Black-Scholes Model tends to overvalue deep out-of-the-money calls and undervalue deep in-the-money calls.

6. The Black-Scholes Model tends to misprice options that involve high-dividend stocks.

To deal with these limitations, a Black-Scholes variant known as ARCH, Autoregressive Conditional Heteroskedasticity, was developed. This variant replaces constant volatility with stochastic (random) volatility. A number of different models have been developed all incorporating ever more complex models of volatility. However, despite these known limitations, the classic Black-Scholes model is still the most popular with options traders today due to its simplicity.

Steps in calculation of BOPM:

1. Find out the value of **t** in terms of years. **For example**, for a call option of 6 months, t = 0.5; for a call option of 73days, t = 73÷365 = 0.2
2. Find the value of **rt** by multiplying the rate of interest with the **t**
3. Find the values of $\mathbf{d_1}$ and $\mathbf{d_2}$
4. Find out the values of $\mathbf{N\ (d_1)}$ and $\mathbf{N\ (d_2)}$ with the help of Area under Normal Curve table
5. Find out the **value of a call option** using Equation

Example: The share of FM Ltd. is currently sold for Rs. 60. There is a call option available at strike price of Rs. 56 for a period of 6 months. Find out the value of call option given that the rate of interest of the investor is 14% and the standard deviation of the return is 30%. Use Black and Scholes model.

Solution:

Given Current Market Price (S) = Rs. 60

Strike Price (E) = Rs. 56

time to expiry (t) = 6 months i.e. $\frac{6}{12} = 0.5$

Annualised Standard deviation (σ) = 30%

in decimal form i.e. 0.30

risk-free interest rate (r) = 14% i.e. 0.14

Step 1 → Value of t = 0.5

Step 2 → Value of rt = 0.14 × 0.5 = 0.07

Step 3 → Calculation of d_1 & d_2

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + (r + 0.5\sigma^2)t}{\sigma\sqrt{t}}$$

$$= \frac{\ln\left(\frac{60}{56}\right) + \left(0.14 + 0.5(0.30)^2\right)0.5}{0.30\sqrt{0.5}}$$

$$= \frac{\ln(1.071) + \left(0.14 + 0.5(0.09)\right)0.5}{0.30 \times 0.707}$$

② [Values are rounded off in table]

ln (1.071) → refer natural logarithm table at the end of the solution.

$$d_1 = \frac{0.0687 + (0.14 + 0.045)\,0.5}{0.2121}$$

$$= \frac{0.0687 + (0.185)\,0.5}{0.2121}$$

$$= \frac{0.0687 + 0.0925}{0.2121} = \frac{0.1612}{0.2121}$$

$$\therefore d_1 = 0.760$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

$$= 0.760 - 0.2121$$

$$= 0.5479, \text{ i.e } 0.55$$ [After decimal take 2 numbers]

$$\therefore d_2 = 0.55$$

Step 4 :→ Find out the values of $N(d_1)$ & $N(d_2)$ with the help of Area under Normal curve table.

(3)

The Values of $N(d_1)$ & $N(d_2)$ represent the Cumulative probabilities that the Standard normal Variable will assume for Values less than d_1 & d_2 respectively. Using Statistical terminology, the cumulative probability of 0 is 50%. or $N(0) = .50$.

The cumulative probabilities for different values of d_1 & d_2 can be found with the help of Area Under Normal Curve Table, given at the end of the solution.

Now $N(d_1) = N(0.760)$

$= 0.5 + 0.2764$ [Value from table]

$= 0.7764$

$N(d_2) = N(0.55) = 0.5 + 0.2088$ [Value from table]

$= 0.7088$

$N \rightarrow$ to be taken as 0.5, because normal distribution curve i.e. will be equal to 1 So Normal distribution function will be 50% i.e. 0.5

Step 5 → find Value of a call option by equation

$$C = S \times N(d_1) - K \cdot e^{-rt} \times N(d_2)$$

$$= 60(0.7764) - 56 \times e^{-0.07} \times 0.7088$$

$$= 46.584 - 56 \times 0.93239 \times 0.7088$$

[refer $e^{-0.07}$ value from the table at the end of the solution]

$$= 46.584 - 37 = ₹9.58$$

∴ Using Black and Scholes model the value of a call option is ₹ 9.58

Natural Logarithms

ln	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1.0	0.0000	0.0100	0.0198	0.0296	0.0392	0.0488	0.0583	0.0677	0.0770	0.0862	10	19	29	38	48	57	67	76	86
1.1	0.0953	0.1044	0.1133	0.1222	0.1310	0.1398	0.1484	0.1570	0.1655	0.1740	9	17	26	35	44	52	61	70	78
1.2	0.1823	0.1906	0.1989	0.2070	0.2151	0.2231	0.2311	0.2390	0.2469	0.2546	8	16	24	32	40	48	56	64	72
1.3	0.2624	0.2700	0.2776	0.2852	0.2927	0.3001	0.3075	0.3148	0.3221	0.3293	7	14	22	30	37	44	52	59	67
1.4	0.3365	0.3436	0.3507	0.3577	0.3646	0.3716	0.3784	0.3853	0.3920	0.3988	7	13	21	28	35	41	48	55	62
1.5	0.4055	0.4121	0.4187	0.4253	0.4318	0.4383	0.4447	0.4511	0.4574	0.4637	6	12	19	26	32	39	45	52	58
1.6	0.4700	0.4762	0.4824	0.4886	0.4947	0.5008	0.5068	0.5128	0.5188	0.5247	6	11	18	24	30	36	42	48	55
1.7	0.5306	0.5365	0.5423	0.5481	0.5539	0.5596	0.5653	0.5710	0.5766	0.5822	6	11	17	24	29	34	40	46	51
1.8	0.5878	0.5933	0.5988	0.6043	0.6098	0.6152	0.6206	0.6259	0.6313	0.6366	5	10	16	22	27	32	38	43	49
1.9	0.6419	0.6471	0.6523	0.6575	0.6627	0.6678	0.6729	0.6780	0.6831	0.6881	5	10	15	20	26	31	36	41	46
ln	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
2.0	0.6931	0.6981	0.7031	0.7080	0.7129	0.7178	0.7227	0.7275	0.7324	0.7372	5	10	15	20	24	29	34	39	44
2.1	0.7419	0.7467	0.7514	0.7561	0.7608	0.7655	0.7701	0.7747	0.7793	0.7839	5	9	14	19	23	28	33	37	42
2.2	0.7885	0.7930	0.7975	0.8020	0.8065	0.8109	0.8154	0.8198	0.8242	0.8286	4	9	13	18	22	27	31	36	40
2.3	0.8329	0.8372	0.8416	0.8459	0.8502	0.8544	0.8587	0.8629	0.8671	0.8713	4	9	13	17	21	26	30	34	38
2.4	0.8755	0.8796	0.8838	0.8879	0.8920	0.8961	0.9002	0.9042	0.9083	0.9123	4	8	12	16	20	24	29	33	37
2.5	0.9163	0.9203	0.9243	0.9282	0.9322	0.9361	0.9400	0.9439	0.9478	0.9517	4	8	12	16	20	24	27	31	35
2.6	0.9555	0.9594	0.9632	0.9670	0.9708	0.9746	0.9783	0.9821	0.9858	0.9895	4	8	11	15	19	23	26	30	34
2.7	0.9933	0.9969	1.0006	1.0043	1.0080	1.0116	1.0152	1.0188	1.0225	1.0260	4	7	11	15	18	22	25	28	33
2.8	1.0296	1.0332	1.0367	1.0403	1.0438	1.0473	1.0508	1.0543	1.0578	1.0613	4	7	11	14	18	21	25	28	32
2.9	1.0647	1.0682	1.0716	1.0750	1.0784	1.0818	1.0852	1.0886	1.0919	1.0953	3	7	10	14	17	20	24	27	31
ln	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
3.0	1.0986	1.1019	1.1053	1.1086	1.1119	1.1151	1.1184	1.1217	1.1249	1.1282	3	7	10	13	16	20	23	26	30
3.1	1.1314	1.1346	1.1378	1.1410	1.1442	1.1474	1.1506	1.1537	1.1569	1.1600	3	6	10	13	16	19	22	25	29
3.2	1.1632	1.1663	1.1694	1.1725	1.1756	1.1787	1.1817	1.1848	1.1878	1.1909	3	6	9	12	15	18	22	25	28
3.3	1.1939	1.1969	1.2000	1.2030	1.2060	1.2090	1.2119	1.2149	1.2179	1.2208	3	6	9	12	15	18	21	24	27
3.4	1.2238	1.2267	1.2296	1.2326	1.2355	1.2384	1.2413	1.2442	1.2470	1.2499	3	6	9	12	15	17	20	23	26
3.5	1.2528	1.2556	1.2585	1.2613	1.2641	1.2669	1.2698	1.2726	1.2754	1.2782	3	6	8	11	14	17	20	23	25
3.6	1.2809	1.2837	1.2865	1.2892	1.2920	1.2947	1.2975	1.3002	1.3029	1.3056	3	5	8	11	14	16	19	22	25
3.7	1.3083	1.3110	1.3137	1.3164	1.3191	1.3218	1.3244	1.3271	1.3297	1.3324	3	5	8	11	13	16	19	21	24
3.8	1.3350	1.3376	1.3403	1.3429	1.3455	1.3481	1.3507	1.3533	1.3558	1.3584	3	5	8	10	13	16	18	21	23
3.9	1.3610	1.3635	1.3661	1.3686	1.3712	1.3737	1.3762	1.3788	1.3813	1.3838	3	5	8	10	13	15	18	20	23
ln	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
4.0	1.3863	1.3888	1.3913	1.3938	1.3962	1.3987	1.4012	1.4036	1.4061	1.4085	2	5	7	10	12	15	17	20	22
4.1	1.4110	1.4134	1.4159	1.4183	1.4207	1.4231	1.4255	1.4279	1.4303	1.4327	2	5	7	10	12	14	17	19	22
4.2	1.4351	1.4375	1.4398	1.4422	1.4446	1.4469	1.4493	1.4516	1.4540	1.4563	2	5	7	9	12	14	16	19	21
4.3	1.4586	1.4609	1.4633	1.4656	1.4679	1.4702	1.4725	1.4748	1.4770	1.4793	2	5	7	9	12	14	16	18	21
4.4	1.4816	1.4839	1.4861	1.4884	1.4907	1.4929	1.4951	1.4974	1.4996	1.5019	2	5	7	9	11	14	16	18	20
4.5	1.5041	1.5063	1.5085	1.5107	1.5129	1.5151	1.5173	1.5195	1.5217	1.5239	2	4	7	9	11	13	15	18	20
4.6	1.5261	1.5282	1.5304	1.5326	1.5347	1.5369	1.5390	1.5412	1.5433	1.5454	2	4	6	9	11	13	15	17	19
4.7	1.5476	1.5497	1.5518	1.5539	1.5560	1.5581	1.5602	1.5623	1.5644	1.5665	2	4	6	8	11	13	15	17	19
4.8	1.5686	1.5707	1.5728	1.5748	1.5769	1.5790	1.5810	1.5831	1.5851	1.5872	2	4	6	8	10	12	14	16	19
4.9	1.5892	1.5913	1.5933	1.5953	1.5974	1.5994	1.6014	1.6034	1.6054	1.6074	2	4	6	8	10	12	14	16	18
ln	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
5.0	1.6094	1.6114	1.6134	1.6154	1.6174	1.6194	1.6214	1.6233	1.6253	1.6273	2	4	6	8	10	12	14	16	18
5.1	1.6292	1.6312	1.6332	1.6351	1.6371	1.6390	1.6409	1.6429	1.6448	1.6467	2	4	6	8	10	12	14	16	18
5.2	1.6487	1.6506	1.6525	1.6544	1.6563	1.6582	1.6601	1.6620	1.6639	1.6658	2	4	6	8	10	11	13	15	17
5.3	1.6677	1.6696	1.6715	1.6734	1.6752	1.6771	1.6790	1.6808	1.6827	1.6845	2	4	6	7	9	11	13	15	17
5.4	1.6864	1.6882	1.6901	1.6919	1.6938	1.6956	1.6974	1.6993	1.7011	1.7029	2	4	5	7	9	11	13	15	17
5.5	1.7047	1.7066	1.7084	1.7102	1.7120	1.7138	1.7156	1.7174	1.7192	1.7210	2	4	5	7	9	11	13	14	16
5.6	1.7228	1.7246	1.7263	1.7281	1.7299	1.7317	1.7334	1.7352	1.7370	1.7387	2	4	5	7	9	11	12	14	16
5.7	1.7405	1.7422	1.7440	1.7457	1.7475	1.7492	1.7509	1.7527	1.7544	1.7561	2	3	5	7	9	10	12	14	16
5.8	1.7579	1.7596	1.7613	1.7630	1.7647	1.7664	1.7681	1.7699	1.7716	1.7733	2	3	5	7	9	10	12	14	15
5.9	1.7750	1.7766	1.7783	1.7800	1.7817	1.7834	1.7851	1.7867	1.7884	1.7901	2	3	5	7	8	10	12	13	15
ln	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
6.0	1.7918	1.7934	1.7951	1.7967	1.7984	1.8001	1.8017	1.8034	1.8050	1.8066	2	3	5	7	8	10	12	13	15
6.1	1.8083	1.8099	1.8116	1.8132	1.8148	1.8165	1.8181	1.8197	1.8213	1.8229	2	3	5	6	8	10	11	13	15
6.2	1.8245	1.8262	1.8278	1.8294	1.8310	1.8326	1.8342	1.8358	1.8374	1.8390	2	3	5	6	8	10	11	13	14
6.3	1.8405	1.8421	1.8437	1.8453	1.8469	1.8485	1.8500	1.8516	1.8532	1.8547	2	3	5	6	8	9	11	13	14
6.4	1.8563	1.8579	1.8594	1.8610	1.8625	1.8641	1.8656	1.8672	1.8687	1.8703	2	3	5	6	8	9	11	12	14
6.5	1.8718	1.8733	1.8749	1.8764	1.8779	1.8795	1.8810	1.8825	1.8840	1.8856	2	3	5	6	8	9	11	12	14
6.6	1.8871	1.8886	1.8901	1.8916	1.8931	1.8946	1.8961	1.8976	1.8991	1.9006	2	3	5	6	8	9	11	12	14
6.7	1.9021	1.9036	1.9051	1.9066	1.9081	1.9095	1.9110	1.9125	1.9140	1.9155	1	3	4	6	7	9	10	12	13
6.8	1.9169	1.9184	1.9199	1.9213	1.9228	1.9242	1.9257	1.9272	1.9286	1.9301	1	3	4	6	7	9	10	12	13
6.9	1.9315	1.9330	1.9344	1.9359	1.9373	1.9387	1.9402	1.9416	1.9430	1.9445	1	3	4	6	7	9	10	12	13
ln	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
7.0	1.9459	1.9473	1.9488	1.9502	1.9516	1.9530	1.9544	1.9559	1.9573	1.9587	1	3	4	6	7	9	10	11	13
7.1	1.9601	1.9615	1.9629	1.9643	1.9657	1.9671	1.9685	1.9699	1.9713	1.9727	1	3	4	6	7	8	10	11	13
7.2	1.9741	1.9755	1.9769	1.9782	1.9796	1.9810	1.9824	1.9838	1.9851	1.9865	1	3	4	6	7	8	10	11	12
7.3	1.9879	1.9892	1.9906	1.9920	1.9933	1.9947	1.9961	1.9974	1.9988	2.0001	1	3	4	5	7	8	10	11	12
7.4	2.0015	2.0028	2.0042	2.0055	2.0069	2.0082	2.0096	2.0109	2.0122	2.0136	1	3	4	5	7	8	9	11	12
7.5	2.0149	2.0162	2.0176	2.0189	2.0202	2.0215	2.0229	2.0242	2.0255	2.0268	1	3	4	5	7	8	9	11	12
7.6	2.0281	2.0295	2.0308	2.0321	2.0334	2.0347	2.0360	2.0373	2.0386	2.0399	1	3	4	5	6	8	9	10	12
7.7	2.0412	2.0425	2.0438	2.0451	2.0464	2.0477	2.0490	2.0503	2.0516	2.0528	1	3	4	5	6	8	9	10	12
7.8	2.0541	2.0554	2.0567	2.0580	2.0592	2.0605	2.0618	2.0631	2.0643	2.0656	1	3	4	5	6	8	9	10	11
7.9	2.0669	2.0681	2.0694	2.0707	2.0719	2.0732	2.0744	2.0757	2.0769	2.0782	1	3	4	5	6	8	9	10	11
ln	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
8.0	2.0794	2.0807	2.0819	2.0832	2.0844	2.0857	2.0869	2.0882	2.0894	2.0906	1	3	4	5	6	7	9	10	11
8.1	2.0919	2.0931	2.0943	2.0956	2.0968	2.0980	2.0992	2.1005	2.1017	2.1029	1	2	4	5	6	7	9	10	11
8.2	2.1041	2.1054	2.1066	2.1078	2.1090	2.1102	2.1114	2.1126	2.1138	2.1150	1	2	4	5	6	7	9	10	11
8.3	2.1163	2.1175	2.1187	2.1199	2.1211	2.1223	2.1235	2.1247	2.1258	2.1270	1	2	4	5	6	7	8	10	11
8.4	2.1282	2.1294	2.1306	2.1318	2.1330	2.1342	2.1353	2.1365	2.1377	2.1389	1	2	4	5	6	7	8	9	11
8.5	2.1401	2.1412	2.1424	2.1436	2.1448	2.1459	2.1471	2.1483	2.1494	2.1506	1	2	4	5	6	7	8	9	11
8.6	2.1518	2.1529	2.1541	2.1552	2.1564	2.1576	2.1587	2.1599	2.1610	2.1622	1	2	3	5	6	7	8	9	10
8.7	2.1633	2.1645	2.1656	2.1668	2.1679	2.1691	2.1702	2.1713	2.1725	2.1736	1	2	3	5	6	7	8	9	10
8.8	2.1748	2.1759	2.1770	2.1782	2.1793	2.1804	2.1815	2.1827	2.1838	2.1849	1	2	3	5	6	7	8	9	10
8.9	2.1861	2.1872	2.1883	2.1894	2.1905	2.1917	2.1928	2.1939	2.1950	2.1961	1	2	3	4	6	7	8	9	10
ln	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
9.0	2.1972	2.1983	2.1994	2.2006	2.2017	2.2028	2.2039	2.2050	2.2061	2.2072	1	2	3	4	6	7	8	9	10
9.1	2.2083	2.2094	2.2105	2.2116	2.2127	2.2138	2.2148	2.2159	2.2170	2.2181	1	2	3	4	6	7	8	9	10
9.2	2.2192	2.2203	2.2214	2.2225	2.2235	2.2246	2.2257	2.2268	2.2279	2.2289	1	2	3	4	5	6	8	9	10
9.3	2.2300	2.2311	2.2322	2.2332	2.2343	2.2354	2.2364	2.2375	2.2386	2.2396	1	2	3	4	5	6	7	9	10
9.4	2.2407	2.2418	2.2428	2.2439	2.2450	2.2460	2.2471	2.2481	2.2492	2.2502	1	2	3	4	5	6	7	8	10
9.5	2.2513	2.2523	2.2534	2.2544	2.2555	2.2565	2.2576	2.2586	2.2597	2.2607	1	2	3	4	5	6	7	8	9
9.6	2.2618	2.2628	2.2638	2.2649	2.2659	2.2670	2.2680	2.2690	2.2701	2.2711	1	2	3	4	5	6	7	8	9
9.7	2.2721	2.2732	2.2742	2.2752	2.2762	2.2773	2.2783	2.2793	2.2803	2.2814	1	2	3	4	5	6	7	8	9
9.8	2.2824	2.2834	2.2844	2.2854	2.2865	2.2875	2.2885	2.2895	2.2905	2.2915	1	2	3	4	5	6	7	8	9
9.9	2.2925	2.2935	2.2946	2.2956	2.2966	2.2976	2.2986	2.2996	2.3006	2.3016	1	2	3	4	5	6	7	8	9
10.0	2.3026																		

ln·[1·071]

under 1.0 row & 7th Column

1·07 = 0·0677

Then see under mean differences it is rounded off = 10

under mean difference of 1 it is 10

So total ln Value is

1·07 = 0·0677

1 = 10

1·071 = 0·0687

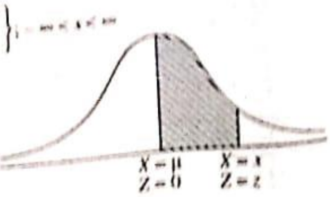
TABLE VI : AREAS UNDER STANDARD NORMAL PROBABILITY CURVE

Normal Probability curve is given by : $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}; -\infty < x < \infty$

and standard normal probability curve is given by :

$$f(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}z^2\right), -\infty < z < \infty$$

where $$Z = \frac{X - E(X)}{\sigma_x} = \frac{X - \mu}{\sigma} \sim N(0, 1)$$



The following table gives the shaded area in the diagram, viz. $P(0 < Z < z)$ for different values of z.

AREAS UNDER STANDARD NORMAL PROBABILITY CURVE

z \ z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

Table E: Continuous Compounding of Re 1 e^x and Continuous Discounting of Re 1

$(e^x): \lim_{m \to \infty}\left(1+\frac{i}{m}\right)^{(nm)}$ or $e^{(i)(n)}$

x	e^x Value	e^{-x} Value	x	e^x Value	e^{-x} Value	x	e^x Value	e^{-x} Value
0.00	1.0000	1.00000	0.45	1.5683	.63763	0.90	2.4596	.40657
0.01	1.0110	0.99005	0.46	1.5841	.63128	0.91	2.4843	.40252
0.02	1.0202	.98020	0.47	1.6000	.62500	0.92	2.5093	.39852
0.03	1.0305	.97045	0.48	1.6161	.61878	0.93	2.5345	.39455
0.04	1.0408	.96079	0.49	1.6323	.61263	0.94	2.5600	.39063
0.05	1.0513	.95123	0.50	1.6487	.60653	0.95	2.5857	.38674
0.06	1.0618	.94176	0.51	1.6653	.60050	0.96	2.6117	.38298
0.07	1.0725	.93239	0.52	1.6820	.59452	0.97	2.6379	.37908
0.08	1.0833	.92312	0.53	1.6989	.58860	0.98	2.6645	.37531
0.09	1.0942	.91393	0.54	1.7160	.58275	0.99	2.6912	.37158
0.10	1.1052	.90484	0.55	1.7333	.57695	1.00	2.7183	.36788
0.11	1.1163	.89583	0.56	1.7507	.57121	1.20	3.3201	.30119
0.12	1.1275	.88692	0.57	1.7683	.56553	1.30	3.6693	.27253
0.13	1.1388	.87809	0.58	1.7860	.55990	1.40	4.0552	.24660
0.14	1.1503	.86936	0.59	1.8040	.55433	1.50	4.4817	.22313
0.15	1.1618	.86071	0.60	1.8221	.54881	1.60	4.95[illegible]	[illegible]
0.16	1.1735	.85214	0.61	1.8404	.54335	1.70	5.4739	.18268
0.17	1.1853	.84366	0.62	1.8589	.53794	1.80	6.0496	.16530
0.18	1.1972	.83527	0.63	1.8776	.53259	1.90	6.6859	.14957
0.19	1.2092	.82696	0.64	1.9865	.52729	2.00	7.3891	.13534
0.20	1.2214	.81873	0.65	1.9155	.52205	3.00	20.086	.04979
0.21	1.2337	.81058	0.66	1.9348	.51885	4.00	54.598	.01832
0.22	1.2461	.80252	0.67	1.9542	.51171	5.00	148.41	.00674
0.23	1.2586	.79453	0.68	1.9739	.50662	6.00	403.43	.00248
0.24	1.2712	.78663	0.69	1.9937	.50158	7.00	1096.6	.00091
0.25	1.2840	.77880	0.70	2.0138	.49659	8.00	2981.0	.00034
0.26	1.2969	.77105	0.71	2.0340	.49164	9.00	8103.1	.00012
0.27	1.3100	.76338	0.72	2.0544	.48675	10.00	22026.5	.00005
0.28	1.3231	.75578	0.73	2.0751	.48191			
0.29	1.3364	.74826	0.74	2.0959	.47711			
0.30	1.3499	.74082	0.75	2.1170	.47237			
0.31	1.3634	.73345	0.76	2.1383	.46767			
0.32	1.3771	.72615	0.77	2.1598	.46301			
0.33	1.3910	.71892	0.78	2.1815	.45841			
0.34	1.4049	.71177	0.79	2.2034	.45384			
0.35	1.4191	.70569	0.80	2.2255	.44933			
0.36	1.4333	.69768	0.81	2.2479	.44486			
0.37	1.4477	.69073	0.82	2.2705	.44043			
0.38	1.4623	.68386	0.83	2.2933	.43605			
0.39	1.4770	.67707	0.84	2.3164	.43171			
0.40	1.4918	.67032	0.85	2.3396	.42741			
0.41	1.5068	.66365	0.86	2.3632	.42316			
0.42	1.5220	.65705	0.87	2.3869	.41895			
0.43	1.5373	.65051	0.88	2.4109	.41478			
0.44	1.5527	.64404	0.89	2.4351	.41066			